# On the string-inspired approach to QED in external field

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## Abstract

Strassler's formulation of the string-derived Bern-Kosower formalism is extended to consider QED processes in homogeneous constant external field. A compact expression for the contribution of the one-loop with arbitrary number of external photon lines is given for scalar QED. Extension to spinor QED is shortly discussed.

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Four years ago Bern and Kosower [1] used string theory to derive a new formalism for the calculation of one-loop amplitudes in field theory. Inspired by this work Strassler derived a set of rules for computing one-loop Green functions in field theory directly from one-dimensional path integrals for relativistic point-particle models [2] . Earlier similar results were obtained by Polyakov [**?**]. The method has been used and extended by several authors so as to allow for example the calculation of effective actions [3] and of higher loops [4] . Also Lam [5] showed that expressions similar to Bern-Kosower rules can be obtained using the Feynman-parameter representation even for multiloop diagrams in QED.

Here we wish to extend this method to study QED processes in external constant electromagnetic fields. We will consider processes described by one-loop diagrams with N external photons. For example the N=2 case will give us the polarization operator [6,7], N=3 case describe photon splitting in external electromagnetic field [8]. Since nonperturbative calculation in $F_{\mu\nu}$ has to be employed , the calculations with external fields are highly nontrivial. For the comprehensive presentation of QED with external fields and the technique used see for example [9].

We will begin with modified Strassler's expression for amplitude with external field $A_\mu(x)$ added

$$\begin{aligned}\Gamma_N\left(k_1,...,k_N\right) = -(i\ g)^N \int\limits_0^\infty \tfrac{dT}{T}\ \mathcal{N}\ \int \mathcal{D}x \exp\left\{-\int d\tau\left[\tfrac{\dot{x}^2}{2\mathcal{E}} - igA(x)\cdot\dot{x} - m^2\right]\right\} \times \\ \times \prod_{i=1}^N \int\limits_0^T dt_i \varepsilon_i\cdot\dot{x}(t_i)\ e^{i\,k_i\cdot x(t_i)}\end{aligned} \tag{1}$$

To calculate this expectation value we will use standard path integral methods of string perturbation theory [10].We will get

$$\begin{aligned}\Gamma_N\left(k_1,...,k_N\right) = -(i\ g)^N \int\limits_0^\infty \tfrac{dT}{T} \prod_{i=1}^N \int\limits_0^T dt_i\ \mathcal{N}\ \int \mathcal{D}x \exp\left\{-\int\limits_0^T d\tau\left[\tfrac{\dot{x}^2}{2\mathcal{E}} - igA(x)\cdot\dot{x} - m^2\right]\right\} \times \\ \exp\left\{+\int\limits_0^T d\tau\ J_\mu(\tau) x^\mu\left(\tau\right)\right\}\Bigg|_{linear\ in\ each\ \varepsilon}\end{aligned} \tag{2}$$

where the source for $x^\mu$ is

$$J^\mu(\tau) = \sum_{i=1}^N \delta\left(\tau - t_i\right)\left(\varepsilon_i^\mu \frac{\partial}{\partial t_i} + i\ k_i^\mu\right) \tag{3}$$

We take $A_\mu(x) = -\frac{1}{2}F_{\mu\nu}x^\nu$ then integration over $x(\tau)$ gives

$$\begin{array}{c} \Gamma_N(k_1,...,k_N) = -(i\ g)^N \int\limits_0^\infty \frac{dT}{T} \prod_{i=1}^N \int\limits_0^T dt_i [2\pi\mathcal{E}T]^{-2} \frac{\left|-\frac{1}{4}\partial^2\right|^{\frac{1}{2}}}{\left|-\frac{1}{4}\partial^2 - \frac{1}{2}igF\otimes\partial\right|^{\frac{1}{2}}} \times \\ \exp\left\{-\frac{1}{2}\int\limits_0^T d\tau \int\limits_0^T d\tau' J^\mu(\tau) G_{\mu\nu}(\tau,\tau') J^\nu(\tau') + \frac{\mathcal{E}}{2}m^2T\right\} \end{array} \tag{4}$$

The matrix Green function satisfies the equation

$$\frac{1}{\mathcal{E}}\ddot{G} + igF\dot{G} = \delta(\tau-\tau') - \frac{1}{T} \tag{5}$$

which has the solution when the condition of periodicity in $\tau \to \tau + T$ is imposed

$$G_{\mu\nu}(\tau,\tau') = \begin{array}{ll} -i\frac{e^{-ig\mathcal{E}F(\tau-\tau')}-1}{gF(e^{-ig\mathcal{E}FT}-1)} + \frac{i}{gFT}(\tau-\tau') & at \quad \tau > \tau' \\ -i\frac{e^{-ig\mathcal{E}F(T+\tau-\tau')}-1}{gF(e^{-ig\mathcal{E}FT}-1)} + \frac{i}{gFT}(\tau-\tau') + \frac{i}{gF} & at \quad \tau < \tau' \end{array} \tag{6}$$

In limit F=0 it give the old Green function of [2]. Using now for determinants results of [3] and choosing the gauge $\mathcal{E} = 2$ we obtain

$$\begin{array}{c} \Gamma_N(k_1,...,k_N) = -(i\ g)^N \int\limits_0^\infty \frac{dT}{(4\pi)^2T} \frac{g^2 a\, b}{\sin(igbT)\sinh(igaT)} \prod_{i=1}^N \int\limits_0^T dt_i \times \\ \exp\left\{-\frac{1}{2}\int\limits_0^T d\tau \int\limits_0^T d\tau' J^\mu(\tau) G_{\mu\nu}(\tau,\tau') J^\nu(\tau') + m^2T\right\} \end{array} \tag{7}$$

where

$$\begin{array}{c} a^2 = (\mathcal{F}^2 + \mathcal{J}^2)^{\frac{1}{2}} + \mathcal{F} \ \ , b^2 = (\mathcal{F}^2 + \mathcal{J}^2)^{\frac{1}{2}} - \mathcal{F} \\ \mathcal{F} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}, \ \ \mathcal{J} = -\frac{1}{4}F^*_{\mu\nu}F^{\mu\nu} \end{array} \tag{8}$$

Now we have as a final result

$$\begin{array}{c} \Gamma_N(k_1,...,k_N) = -(i\ g)^N \int\limits_0^\infty \frac{dT}{(4\pi)^2T} \frac{g^2 a\, b}{\sin(igbT)\sinh(igaT)} e^{m^2T} \prod_{i=1}^N \int\limits_0^T dt_i \times \\ \exp\frac{1}{2}\sum_{i,j}^N \left\{k_i G(t_i-t_j)k_j - i\varepsilon_i \frac{\partial}{\partial t_i}G(t_i-t_j)k_j - ik_i\frac{\partial}{\partial t_j}G(t_i-t_j)\varepsilon_j - \varepsilon_i\frac{\partial^2}{\partial t_i \partial t_j}G(t_i-t_j)\varepsilon_j\right\}\Big|_{linear\ in\ each\ \varepsilon} \end{array} \tag{9}$$

where due to translation invariance the integration over N +1 variables is really over N variables.Here for symmetry we does not remove this redundant integration though this can be made at any step of calculation. In the case N=2 eq.(9) reproduce results of [7].

Extension to spinor case is straightforward : we need only new additional Green function $G_F$ [2] which now obey

$$\dot{G}_F + ig\mathcal{E}FG_F = 2\delta(\tau - \tau') \tag{10}$$

which has a solution when the condition of antiperiodicity in $\tau \to \tau + T$ is imposed

$$G_F\left(\tau, \tau'\right) = \begin{array}{cc} 2\frac{e^{-ig\mathcal{E}F(\tau-\tau')}}{e^{-ig\mathcal{E}FT}+1} & at \quad \tau > \tau' \\ 2\frac{e^{-ig\mathcal{E}F(T+\tau-\tau')}}{e^{-ig\mathcal{E}FT}-1} & at \quad \tau < \tau' \end{array} \tag{11}$$

Note finally that calculations with functions of the matrix $F$ can be easily done using techniques presented in the appendix of paper [11].

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